

BOOK REVIEWS

**Moktefi, Amirouche & Abeles, Francine F., eds. (2016),
'What the Tortoise Said to Achilles'. Lewis Carroll's Paradox of
Inference, special double issue of *The Carrollian*, *The Lewis Carroll
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There is a remarkable similarity between the works of Kurt Gödel and Charles Dodgson, perhaps better known to the general audience as Lewis Carroll. Both had this cunning ability to write short papers, one, two pages maximum, that were easily and quickly read and then made you think for years to come. This special double (!) issue of *The Carrollian*, *The Lewis Carroll Journal*, is entirely devoted to such a short 'story' of Carroll, entitled 'What the Tortoise Said to Achilles' (WTSA from here on), published in *Mind* in 1895. In the A5-format of *The Carrollian* it takes up three pages. (So one is entitled to deduce that the paper is part of this issue.) The odd thing is that what the problem is, is easily explained but how to deal with it, seems to be the deep issue. As the editors write in their introduction: "What is more remarkable is that in the articles that have appeared in journals and books for over 120 years, there has been no accepted resolution to the problem Carroll posed in WTSA." (p. 2) And they should know for the editors are Amirouche Moktefi and Francine F. Abeles. The former is lecturer in Philosophy at Tallinn University of Technology in Estonia, having obtained his PhD in Strasbourg in 2007 and, most important of all, an expert on matters Carrollian. The latter is professor emerita at Kean University in Union, NJ, USA. She too is an expert in history of logic, especially the period wherein WTSA is situated. So, yes, indeed they should know.

What then is the problem, also known as Lewis Carroll's Paradox of Inference? Suppose we have a logical proof that shows that from some premises A_1, A_2, \dots, A_n a conclusion B follows. There are two options to doubt the truth of B . On the one hand because not all premises are true or on the other hand one disputes the logical rule that allows the transition from the premises to the conclusion. In the former case there need not be a problem: the proof itself can still stand even though one does not accept the conclusion. If I prove that from the premises 'I am a bird' and 'All birds can fly' the conclusion necessarily follows that I can fly, then surely this is correct although I should better not try

to jump out of the window. But in the latter case the question must be raised: how could one be convinced that the rule is acceptable? And here Carroll's reply is that it certainly does not help to include (a verbal expression of) the rule among the premises for that only begs the question: why should one accept the conclusion of a similar argument, augmented with an additional premise? If one is not careful, one is trapped in a *regressus ad infinitum* and that precisely happens to poor Achilles and, as Carroll writes, understandably "there was a touch of sadness in his tone".

A different way of formulating the problem is this: suppose you derive B from A. Then one seems entitled to write down the hypothetical 'If A, then B'. We now have three elements: A, B and 'If A, then B'. Question: is the hypothetical statement acceptable, even if A is false? The present-day answer, probably taught in any logic course in the West, is a simple 'yes', because a material implication is always true if the antecedent is false. Not so in Carroll's days. There was a quite interesting dispute going on. In fact, it is related to a lesser known paper by Lewis Carroll, also printed in *Mind* before WTSA in 1894, titled 'A logical paradox'. The central point is whether two statements of the form 'If A, then B' and 'If A, then not B' can be compatible, given that A is false. (Which is precisely what we would conclude today, namely that 'not A' follows from both statements.) The five contributions in this volume all deal with these problems and the end result is quite fascinating and instructive. In fact, nearly all authors seem to agree, though they disagree among themselves—see, for example, footnote 11 on page 87 in Pascal Engel's paper disputing the mistaken interpretation that Mathieu Marion attributes to him—that it is not clear what exactly the problem is that has been raised by Carroll (and so my presentation above therefore needs to be read as a first-order formulation in need of amendment), especially since he himself did not provide an answer. Here is a short survey of the issue.

The first contribution is by the editors themselves: "The Making of 'What the Tortoise Said to Achilles': Lewis Carroll's Logical Investigations Toward a Workable Theory of Hypotheticals". As the title indicates, their focus is first and foremost historical and in fact my historical summary above is largely based on their contribution. One of the important conclusions of their paper is: "Hence, 'A logical paradox' and 'What the Tortoise said to Achilles' were the results of this ongoing investigation and not accidental contributions." (p. 40). This reviewer must confess that he too believed WTSA to be a literary 'folly', my sole argument being that it was published under his literary pseudonym. I stand happily corrected!

Next comes Mathieu Marion's paper, entitled "Lessons from Lewis Carroll's Paradox of Inference". In the first part he focuses on the first reactions to WTSA, involving John Cook Wilson, Gilbert Ryle and Bertrand Russell. An important point that he emphasizes is that the paradox is already present, also in the form of an infinite regress, in Bernard Bolzano's (1837) *Wissenschaftslehre*. In the second part he looks at later developments with a special focus on W. V. O. Quine. And, finally, he also addresses issues in how the history of logic is (mis)presented by authors such as I. M. Bochenski.

In his delightful contribution 'What Did Lewis Carroll Think the Tortoise Said to Achilles?' George Englebretsen presents a personal journey how he changed his mind about what WTSA is all about. One of the important points he emphasizes is that any attempt to understand Carroll's intentions and aims has to take into account the time period or, as Englebretsen expresses it, that "Carroll was a *Victorian* logician" (p. 80). Incidentally, he is the only author in this volume who has already published in *The Carrollian*, to be precise, its forerunner known as *Jabberwocky*—I will not reproduce here the Carrollian reference to this mythical animal—in 1974 and 1994, precisely about WTSA.

The broadest scope is offered by Pascal Engel in his paper "The Philosophical Significance of Carroll's Regress". Four themes are discussed: "(a) the nature of logical inference, (b) the nature of our understanding of logical rules and logical knowledge, (c) the justification of logical rules, (d) the nature of normative guidance in both theoretical and practical reasoning" (p. 84). As one might expect, the famous Wittgensteinian concept of 'rule-following' plays a crucial role and "invites us to draw parallels between logic and ethics, and between epistemology and meta-ethics. It is a litmus test for many of the most interesting issues of contemporary philosophy." (p. 105) The implications of this invitation are really quite serious for it means that to use a 'simple' *modus ponens* in an argument or proof ceases to be an 'innocent' act.

Finally, 'Required by Logic' by John Woods is close in spirit to Englebretsen's paper. What the reader is offered is a close examination of WTSA to understand what is going on in a diagnostic fashion. It is perhaps not surprising if one knows his work that Woods is drawing our attention to the dialogical nature of the presentation. After all, it is one thing that the Tortoise puts forward a particular claim and challenges Achilles to defeat this claim but it is quite another thing to come up with an effective strategy to reach that goal. So, a sequel to WTSA could be WASHRT, 'What Achilles Should Have Replied to the Tortoise'.

It is rather striking that, if one lists the logicians and, by extension, the philosophers mentioned in this volume, one obtains a list of all major twentieth-century contributors, ranging from Bolzano, Russell, Ryle and Braithwaite to Dummett, Wittgenstein, Prior and Quine. Two conclusions can be drawn from this observation. The first, already mentioned, is that the problem that Carroll so casually wrote down in the form of a dialogue deals in fact with deep issues in the philosophy of logic. The second is that agreement is (still) largely lacking, precisely because it involves fundamental processes such as inference, (logical) proof, grasping and applying a rule.

Finally, it must be mentioned that the volume contains two extras: a selective bibliography and the correspondence with George Frederick Stout, the editor of *Mind* at that time. As to the latter, these letters, three in total and published as a whole for the first time in this volume, actually are apparently the only occasion where Carroll expresses some of his thoughts specifically about WTSA and the related 'A logical paradox'. As to the former, the bibliography, put together by one of the editors, Amirouche Moktefi, and research librarian Clare Imholtz, is divided in two sections. The first deals with reprints and translations of WTSA and the second with works devoted to WTSA, not meant to be exhaustive. A somewhat curious feature of this list is that its order is chronological and not by name of author.

In summary, this double issue of *The Carrollian* is a welcome addition to the literature on WTSA that, probably, many among us know because of Hofstadter's (1979) classic, *Gödel, Escher, Bach. An Eternal Golden Braid*. However, the surplus value of this volume is that, as mentioned repeatedly in this review, it shows that far more than just a rather amusing puzzle, it is really about a deep problem in the heart of philosophical logic and thus should merit our full attention.

References

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