Acta Baltica Historiae et Philosophiae Scientiarum
Vol. 6, No. 1 (Spring 2018)
DOI : 10.11590/abhps.2018.1.04

The Genesis of General Relativity: An Inter-Theoretical Context

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Abstract: The aim of the paper is to amend the received view on the genesis and approval of general relativity (GR) owing to the common scientific practice of its functioning, the history of science data and the philosophy of science reasons. The genesis of GR is elucidated as an instance of an epistemological model of a mature theory change that hinges upon 'old' mature theories' encounter and interaction. The arguments are strengthened in favour of the tenet that the dynamic creation of the GR had been continually governed by strong internal tensions between two research traditions, that of special relativity and Newton's gravity. The 1907 encounter of the traditions, their deep interpenetration and subtle intertwinement entailed construction of a vast hybrid domain, at first with an irregular set of theoretical models. Step by step, on consecutive eliminating of the contradictions between the contrived models, the hybrid set was put into order by dint of the 'principle of equivalence'. It is contended that one of the reasons for the GR victory over the ingenious rival programmes of Abraham and Nordström was the synthetic character of Einstein's research programme. As a result of reconciling and amalgamating the 'physical' and 'mathematical' approaches, embodied in Abraham, Einstein and Nordström's crossbred theoretical models, Einstein was able to explain the anomalous motion of Mercury.

Keywords: *Abraham, Einstein, 'Entwurf', general relativity, hybrid models, Nordström*

Introduction

It is proverbial that Albert Einstein's vigorous efforts to create the General Relativity (GR) were accompanied by its rival versions ingeniously conjured up by Gunnar Nordström, Max Abraham, Gustav Mie *et al*. In particular, in 1912–1914, the Finnish mathematician Gunnar Nordström advanced a scalar Lorentz covariant gravitation theory. In a paper submitted to *Physikalische Zeitschrift* in October 1912, Nordström maintained that he had found a rival to Einstein's hypothesis which would "leave c [the speed of light] constant and still adapt the theory of gravitation to the relativity principle in such a way that gravitational and inertial masses are equal" (Nordström, 1912, p. 1126).

Likewise, in 1912, a Gottingen master of classical electrodynamics Max Abraham proposed a host of scalar and vector Lorentz covariant gravitation theories where light and gravitation appeared to have the same speed of propagation. In Abraham's skilful scalar theories "c, the speed of light, depends on the gravitational potential. This hypothesis was first enunciated by Einstein (Ann. d. Physik, 35, 1911, p. 898)" (Abraham, 1912b, p. 793).

The writings of Abraham and Nordström are still taken as whimsical *delusions* capable to stir up problem situations at best and to incite critical discussions around the GR, highlighting all its opulence and splendor. Eventually a received view on the genesis of the GR was set up according to which, in the process of its construction, Einstein decisively "rejected" Lorentz covariant scalar and vector theories of gravity.

However, some current history-of-science insights (Norton, 1992; Renn & Sauer, 2007; Renn, 2007b; van Dongen, 2010) prompt one to take the standard view with a considerable grain of salt. To begin with, the Einstein–Nordström correspondence convincingly underscores that it was Albert Einstein himself who, before November 1915, and even *after* the creation of GR preliminary metric version—the notorious '*Entwurf*' (1913)—took active part in construction of Nordström's scalar relativistic theories. Einstein was in close contact with the Finnish mathematician during the period in which the Nordström theory was advanced. The theory actually advanced through an intensive exchange between Einstein and Nordström, with Einstein often generating the ideas decisive to the evolution of the theory. By and large, the theory might more accurately be called the "Einstein-Nordström theory".

The next apparent example is A. Einstein and A. Fokker's paper published in early 1914 that aimed at an "application of new mathematical methods, used in Einstein and Grossmann's paper, to Nordström's theory" (Einstein & Fokker, 1914). Moreover, in the same paper in early 1914 the substantial connections between Nordström's theory and conformally flat space-times were unfolded. Therefore it comes as no surprise that it was within Nordström's 1912 theory where the gravitational field equation R= $\varkappa$ T ($\varkappa$ = const) was first derived, with R being fully contracted Riemann-Christoffel tensor and T the trace of the stress-energy tensor (in the case of an unstressed, static matter distribution). This field equation is an apparent harbinger of Einstein's illustrious equations presented to *Preussiche Akademie der Wissenschaften* on November 25, 1915 (Einstein, 1915).

Moreover, the consequences of the '*Entwurf*' and the GR coincide with the consequences of the theories of Nordström and Abraham for a number of important cases in certain reasonable approximations. For instance, the '*Entwurf*' is reduced to a theory with a four-vector field potential that is formally analogous to Maxwellian electrodynamics in the so-called 'weak field approximation'. Furthermore, special relativity (SR) turns out to be an inescapable intermediary step in the thorny transition from the GR to Newton's theory of gravitation (see, for instance, Landau & Lifshitz, 1987). But this transition is grounded on the supposition, for weak and stationary gravitational fields, that the gravitational field is described by a scalar in flat (Minkowski) space-time, i.e. on the reduction to scalar Nordström's theory.

Likewise, the so-called 'linear approximation' in GR, still in common use to account for gravitational waves' (Einstein, 1916) propagation and detection (see, for instance, the recent 2015–2017 LIGO experiments), presupposes the transition to such a theory of gravitation in which the gravitational wave, in full analogy with classical electrodynamics, is described by a vector in flat space-time, i.e. the transition to vector theory of Abraham (1915). Abraham's claim that his theory contained Einstein's as a limiting case was rebutted by Einstein, though. But, nevertheless, in both cases the relations between the GR and fine theories of Nordström and Abraham strongly resemble the common pattern of classical electrodynamics where the general potential is represented by many-component object such as a vector or a tensor, which, in the special case of a static field, reduces to a single-component mathematical object.

Furthermore, Einstein's initial approach to the GR creation, on the one hand, and Nordström's and Abraham's approaches, on the other hand, were in many crucial facets ***complementary***. For instance, Einstein's theory of static gravitational field (Einstein, 1912a) was incited by substantially physical considerations based on the equivalence principle, while Abraham's theories started from mathematical considerations related to Minkowski formalism. Yet the diligent investigation of the so-called 'Zurich Notebook' that was employed by Einstein while constructing the '*Entwurf*' theory (where the corresponding entries begin in mid-1912 and end in early 1913) brings some astonishing new light on Einstein's thought laboratory.

The 'Zurich Notebook' has disclosed that Albert Einstein was persistently struggling forward along the so-called '***dual strategy***' that embraced both a physical and mathematical approaches to finding the ultimate field equations (Janssen *et al.*, 2007; Renn & Sauer, 2007; van Dongen, 2010). The 'dual method' suggested that Einstein proceeded first and foremost from a set of constraints of a *physical* nature (Newtonian limit plus conservation of energy and momentum). On the other hand, the complementary 'top-down' (van Dongen) *mathematical* approach originated from the Principle of General Covariance.

The key point is that the 'dual strategy' necessarily employed both physical and mathematical approaches: "it was an *iterative* process that began with trying out one approach, and then checking results with the demands or results of the other approach" (van Dongen, 2011, p. 11). Physics-first prevailed and lead to the '*Entwurf*'; but then the mathematics approach took over and incessantly led to the full-blooded 1915 General Relativity.

Yet it should be stressed here that Einstein's "physical" and "mathematical" strategies, the vacillation between whom brought to '*Entwurf*' and GR poignant construction, were entrenched in two research traditions encountered circa 1907 (Renn & Sauer, 2007, p. 125).

Einstein repeatedly made clear his preference for Nordström's ingenious theory over other artful rivals; for instance, in September 1913 presentation of the '*Entwurf*' theory (see Section 3 for details) to the 85th Congress of the German Natural Scientists and Physicians in Vienna. His single and rather meek critical remark consisted in that the theory was incompatible with Mach's principle—a vice that could turn out a virtue to a Naturforscher biased against metaphysical castles in the air. Later none other than Wolfgang Pauli (1921) christened Nordström's theory an 'empirical blunder' since it had not predicted any deflection of a light ray by a gravitational field and had not explained the anomalous motion of Mercury. Yet there had been no eclipse expeditions in 1913 and Einstein and Grossmann's '*Entwurf*' turned out to be a hangdog incapable of providing a trustworthy explanation of the anomalous motion of Mercury.

On the contrary, in 1912, G. Pavani meticulously calculated the perihelion shift of Mercury according to Abraham's theory, arriving at a value of 14", 52, that is approximately one third of the observed one. Thus, even Abraham's vector theory made a more accurate prediction than the '*Entwurf*'. If one allows that Einstein's discovery (due to Michele Besso's diligent calculations) of the failure of the '*Entwurf*' theory to yield the correct perihelion shift of Mercury was made as early as in the summer 1913, one can conclude that Abraham's advancement was a vigorous spur that prompted Einstein to construct GR out of the '*Entwurf*'.

All the abovementioned hallmarks of the GR genesis, advancement and functioning, and preponderantly ***the common practice of its implementation*** bolster the following conclusions.

(a) The relations between the GR and its ingenious rivals were far more complicated in 1907–1915 than it may seem from the pestered 'truth–falsity' dilemma, so that one can contemplate the interlacement and *interpenetration* of rival 'paradigms' into each other.

(b) Einstein's GR was better than its inimical rivals if only for the reason that it *encompassed them all* in significantly modified forms. (Just as the GR embraces Newton's theory of gravitation and the SR, or just as the Maxwellian electrodynamics encompasses the partial theoretical schemes of Coulomb, Ampére, Biot & Savare *et al.*).

(c) Einstein could complete the reconciliation of the knowledge on gravitation and inertia (represented by classical mechanics) and the knowledge on the structure of space and time (embodied by the SR) via the '*Entwurf*–GR' transition only. As a result of reconciling and amalgamating the 'physical' and 'mathematical' approaches, embodied in Abraham, Einstein and Nordström's crossbred theoretical models, Einstein was able to explain the anomalous motion of Mercury.

Hence the aim of the paper is to amend the standard view on the GR genesis and advancement by taking into account the abovementioned history of science data, philosophical (and sociological) reasons and modern common scientific practice of its functioning. My main idea consists in that one of the reasons for the GR victory over the rival programmes of Abraham and Nordström lay in a synthetic character of the Einstein research programme. Einstein's programme did supersede the rival ones because it did deftly assimilate sober premises of the Nordström programme as well as judicious presuppositions of the programme of Abraham. In particular, Einsteinian programme's convincing victory over its rivals became possible since Einstein had put forward as a basic synthetic principle the principle of equivalence that radically differed from that of rival approaches by its open, flexible and regulative character.

In the second section of the paper, a lucid epistemological model that fits some achievements of current philosophy and history of science and deals with mature theory dynamics and structure is exhibited. The model is a gist of the present study asserting that the history of physics does not advance through the creation *ex nihilo* of new paradigms, but rather through long-term processes of reconciliation, interpenetration and intertwinement of 'old' research traditions.

In the third section of the paper the initial stage of the GR creation (1907–1912) is scrutinized. The crux is a premise that the invention of relativistic theory of gravity had commenced with the crossbred object construction in Einstein's 1907 paper, i.e. with the implantation of mass-energy relation into the theory of gravity. The crossbred object entry—the introduction of inertial and simultaneously gravitational mass—led to a penetration of SR methods into Newtonian theory of gravity and to a reverse penetration of Newtonian gravity methods into the SR. As a result, the both theories were radically rebuilt from within and the corresponding changes in both of them were induced. The changes were epitomized in the specific sequences of crossbred models, the byproducts of the transformation performed.

(i) On the one hand, an inevitable consequence of the relentless penetration of SR into Newtonian theory of gravity turned out to be Nordström's and Abraham's scientific research programmes.

(ii) On the other hand, no less inevitable, owing to the equivalence principle, was the Newtonian theory penetration into the SR. It led to the sequence of Einstein's works on the generalization of relativity principle and to spreading the principle not only on inertial systems of reference, but on the various accelerated systems as well. But the most valuable result of the hybrid theories of Nordström and Abraham consisted in that the both theories maintained some *extremely promising hints* on how the global theory could be created. As a result, the GR emerged as a 'synthetic approach' unifying all the positive achievements of the other alternative approaches. Hence the climax of the stage was Einstein's proposal and apprehension of the equivalence principle that became one of the firm GR heuristic foundations.

The fourth section of the paper (1912–1913) is dedicated to the '*Entwurf*' construction. This metric theory rose not as 'phoenix from the ashes' but via the synthesis of Abraham's and Nordström's theoretical schemes, as well as from the preliminary nonmetric theoretical schemes of Einstein. The staple was the metric tensor introduced owing to the equivalence principle and Nordström, Laue and Planck's startling results.

The fifth section of the paper (1913–1915) grapples with the GR construction out of the '*Entwurf*'. It is exhibited that the main thrust in passing from the '*Entwurf*' to the GR came from Einstein's 'dual strategy' inducing him to *reconcile* physical and mathematical approaches. And only *after* he efficiently reconciled them, i.e. only after he realized the general covariance of the gravitational field equations could he turn to successful explanation of the Mercury perihelion. Hence the decisive impetus in constructing the General Relativity came from the attempts to reconcile physical and mathematical approaches, embodied in Abraham, Nordström and Einstein's crossbred theoretical models. Trustworthy explanation of Mercury perihelion motion appeared to be a lucky by-product of strenuous reconciliation efforts.

A lucid epistemological theory change model

The dynamics of the theory of gravity was predominantly governed by strong *internal tensions*, blatant contradictions *within* the knowledge system rather than by new empirical knowledge, which played only a subordinate role at best. In this section, a corresponding perspicuous epistemological model dealing with mature theory dynamics and structure is posited.

The current philosophy of science discourse on scientific revolutions allows one to elucidate the views on the structure and functioning of scientific theories, on the one hand, and to invent sufficiently sweeping and exact theory change epistemological models, on the other. In particular, according to one of the models (Nugayev, 1999), a scientific revolution is engendered by encounters of some entrenched "old" paradigms, scientific research programmes or research traditions that cannot be reconciled in a common way—by reducing of one of them to another. The way out of the predicament is to work out such a global theory that encompasses all the theories involved in significantly modified forms. The global theory is aimed at eliminating the tensions, smoothing away dissensions between different paradigms involved.

Just to recapitulate Werner Heisenberg's startling *Physik und Philosophie* (1959):

> probably, as a kind of general supposition, it can be said that those directions in the history of human thought appeared to be most fruitful, where different ways of thinking had encountered. These ways of thinking are deeply rooted in different spheres of human culture, or in different times, in different cultural milieu, or in different religious traditions. When they really meet with each other, when they correspond to each other so that an *interaction between them takes place*, one hopes that novel and interesting discoveries will follow" (my italics). (Heisenberg, 1959)

In the course of global theory invention an indispensable preliminary stage shows itself in the construction of a series of hybrid theories. The latter are persistently set up to the climax when such a felicitous hybrid model is constructed that is able to outline the fruitful way of the global model creation through the generalization of models that belong to the lower level of mature theories. According to the aforementioned epistemological model, ***radical breakthroughs in science were not due to ingenious invention of new paradigms or the creation of new ideas ex nihilo, but rather to the long-term processes of the reconciliation and interpenetration of 'old' research traditions preceding such breaks.***

It is a commonplace that no profound epistemological model of scientific revolutions can be established without preliminary elucidating the structure of mature scientific theories. Yet what I want to stress is that a mature theory of 19th and 20th centuries' physics encompasses not a single model or a bundle of models. It embraces a *bundle of groups of models* that are related to one another in rather subtle ways. A mature theory is so structured that the host of its models is disseminated over at least three following interconnected levels (Stepin, 2005; see also the comments in Vihalemm & Müürsepp, 2007).

(1) The level of the ***basic*** ideal model or the level where 'the Fundamental Theoretical Scheme' resides.

(2) The level of the *subordinated* ideal models (or 'the Partial Theoretical Schemes') constructed out of the basic one according to certain (tacit) rules.

(3) The level of the '*Empirical Schemes*' that can be approached through the level of partial theoretical schemes.

For instance, the relations between the basic objects of Newtonian mechanics are set up by Newton's laws. The derivative objects of Newtonian mechanics are 'an absolutely rigid body', 'central field', 'harmonic oscillator', etc. The relationships between them are fixed by certain laws of Newtonian mechanics: that is, by the laws of rigid rotation, movement in the central field, etc.The set of basic objects of a mature physical theory forms the *basis*, i.e. the definite subsystem of theoretical objects. All the basic theoretical objects are apparent idealizations and cannot exist as real bodies. For example, the material point is defined as a body free of dimensions. As for the other basic objects of Newtonian mechanics, it is assumed that an inertial system of reference can be totally isolated from external influence.

The derivative subsystems are *subordinated* (Stepin, 2005) to the basic one, but are independent of each other, referring to different fragments of the same domain of validity. Each subsystem is characterized by its own set of notions and mathematical equations that form a special part (section) of the mature theory. For instance, classical mechanics consists of several independent sections: 'small-oscillations mechanics', 'mechanics of rigid body rotations', 'mechanics of movement in a central field', etc. Each of these sections is characterized by its own subsystem of derivative objects. Each subsystem is a specific model of a particular type of mechanical motion (the small oscillations model, the rigid rotations model, etc.). Relations between the elements of the subsystems are fixed by particular laws of classical mechanics.

In general, the relations between a subsystem of basic objects and a subsystem of derivative ones can be described as follows. Any derivative system is obtained from the basis by a process of reduction. It means that any mature theory is developed not by formal (logical, mathematical) means only, but also through *gedankenexperiments* with abstract theoretical objects. The reduction is put into effect by scrutinizing the features of the empirically fixed domain of validity. This domain can be "seen through" a cognitive lens of an ideal model, formed by correlations of basic objects. According to the peculiarities of each concrete experimental situation, various constraints may be imposed on the system of basic theoretical objects. This enables one to define the system, transforming it into a subsystem of derivative objects. The fundamental equations are then applied to the subsystems of derivative objects. In accordance with the system features, they are transformed into the partial laws. The informal nature of such manipulations converts such an inference into a special problem solving operation. The solutions of such problems are included in a theory at its origin. To a theoretician bothered by applying a theory, they serve as a pattern for subsequent activity. Each problem is solved in accordance with primary 'paradigms' (in Thomas Kuhn's sense).

In the GR, the host of paradigm examples embraces the derivation of the laws of Newton's theory of gravity from Einstein's equations in the so-called 'weak field approximation'. In the case of weak gravitational field such a system of reference is chosen in which all the metric tensor components slightly differ from their Minkowski values: $g = \eta + h$ (see, for instance, Landau & Lifshitz, 1983). The further constraint to ignore the squares and the other multiples of h is necessary for the transition to Newton's theory of gravity. But it means nothing else that index rising operation is carried out by η – the metric tensor of **flat** space-time. As a result, in the weak field approximation the gravitational field equations take the form of usual wave equation in flat space-time for (Nordström's) scalar potential. Thus the basic theoretical object of Nordström's nonmetric scalar theory turns out to be constructed from the GR basis. "This is quite natural since the weak field is considered as a tensor in flat space-time" (Zeldovich & Novikov, 1973, p. 56) and is described by an equation $h_{ik} = -\eta_{ik}\, 2\varphi/c^2$.

The **construction** of derivative objects from the basic ones enables one to compare theoretical knowledge with experience, to explain the results of real experiments. To this end, an empirical equation—an intermediate relation—is derived from the partial law. In this equation the special constructs are introduced. In contrast to abstract objects, the newly born constructs are no longer idealizations and can be compared with real bodies now. These constructs are called *empirical objects*, and their systems—special representations of empirical situations—are called 'empirical schemes'. The empirical objects are not equivalent to real bodies. An empirical object cannot be compared with a single body with which an experimentalist operates, but only with a *class* of such objects. Consequently, an empirical scheme corresponds not to a concrete experimental situation, but to a *type of such situations*. For example, the empirical scheme of the Biot & Savare experiment with a magnetic needle and a conductor refers to any experiment with any current in the conductor and any sufficiently small magnetic needle.

A mature theory becomes an **established** one when the links between all the three levels of the organization are vigorously set up; it makes possible to use the mature theory as an effective instrument for making predictions. The bonds between all the three levels of an established mature theory should be sufficiently rigid ones. Their rigidity allows one to connect a prediction referring to the upper level with all the levels of a mature theory. Hence it allows one to construct an experimental device to check the prediction. A new result, obtained in the course of advancement of mathematical apparatus, influences all the levels of a mature theory immediately. Hence a theory can predict, and the predictions can be verified. A mature theory obtains the status of an established one when at least some of its predictions are posited to be successful. It demonstrates that the system of basic objects is complete, and the links between all the three levels are made robust.

Owing to the complicated structure of a mature theory, the global theory creation appears to be a slow, adamant and consequent ascent from the lower levels up to the top ones. Any transition from the lower level to the upper one is impossible until the construction of all the lower-level models is finished. Yet an important remark here is that the lower models (that served at scaffolding the upper ones) are not eliminated; they can be discovered not only in history-of-science archives. They can be transpired in real practice of theories' functioning (in implicit forms, as a rule). I reckon that the basic models of Nordström's and Abraham's theories constitute partial theoretical schemes of the GR, as well as Pieter Garber's 1898 Mercury perihelion results constitute an empirical scheme of the GR.

Construction of the hybrid models via the equivalence principle

The advent of the special relativity (SR) and the apparent incompatibility between Newton's theory of gravitation and the SR theory perplexed Einstein and his contemporaries with the task of constructing a relativistic theory of gravitation. Apparent *contradictions* between the theories consisted first and foremost in the fact that according to Newton's theory the velocity of gravitational interaction was famously infinite. On the other hand, the SR prohibits the signals travelling faster than light.

It therefore comes as no surprise that it was Einstein's 1907 review 'On the Relativity Principle and the Conclusions Drawn from it', published in Johannes Stark's *Jahrbuch der Radioaktivität und Elektronik*, that laid the true conceptual foundations for relativistic theory of gravity (Einstein, 1907, pp. 254–255). In the fifth part of his epoch-making paper, Einstein formulated first his '*principle* *of equivalence*'. As he later recalled, when he had prepared the 1907 review article for publication, he had tried to modify Newton's gravitational theory so as to reconcile it with the SR. The corresponding attempts had shown that it was possible, but Einstein had relinquished them since they were grounded on physically unacceptable hypotheses (Pais, 1982).The importance of the equivalence principle for the creation of the GR consists in that

> this assumption extends the principle of relativity to the uniformly accelerated translational motion of the reference system. The ***heuristic value*** of this assumption rests on the fact that it permits the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, the latter case being to some extent accessible to theoretical treatment" (Einstein, 2007, p. 302; my bold italics).

Note that Einstein was first and foremost interested not in the ontological, metaphysical content of his principle that could enable him to elevate the tenet up to the status of some Ultimate Law of Nature. The latter would be valid everywhere with any degree of validity being contemplated by a Super Reason trying to grasp the essences of the things and events. (For it is well known, according to Norton, 2007, that in 1907 Einstein was unaware of Eötvös's exact experimental results regarding the equality of inertial and gravitational mass. Moreover, Papapetrou in 1951 found that in the GR a rotating body falls differently, in general, from a non-rotating body).

Thus, Einstein was seeking the *heuristical* components of the principle (see Ryckman, 2005 for details). In gravity purview he strove for comprehending gravitational and inertial phenomena from a single point of view (Janssen, 2012, p. 162).

In my view, it was consequent implication of the equivalence principle that promised to invent a consequence of hybrid models unifying the SR and Newton's theory of gravity. For Einstein the principle of equivalence was not so much a Law of Nature as a ***pattern***, a 'paradigm' for construction of gravitation theories.

In particular, it enabled the investigation of special cases of the gravitational field by means of the study of accelerated motion. So, until 1911 Einstein had committed himself mainly to exploring, by means of the equivalence principle, the effects and conceptual changes characterizing a new theory of gravitation, evidently without seriously attempting its construction. Only in early 1912 was he challenged by the provoking publication of Max Abraham to elaborate such a theory, at least for the special case of a static gravitational field (see Norton, 1986).

On the other hand, the second important component of Einstein's heuristic—"the Lorentz model of a field theory" (Renn & Sauer, 2007)—enabled Einstein to conceive Newtonian gravitation and inertia as special cases of a more general interaction. For the case of uniform acceleration he could directly identify inertial effects with a scalar Newtonian gravitational field and he expected that he would be able to do the same for more general cases by generalizing the notion of gravitational field. A paradigm for the generalizations was of course provided by relativistic electrodynamics. It was Einstein's SR that "unified" electricity and magnetism through treating electric field **E** and magnetic field **B** as different facets of one and the same electromagnetic field tensor $F_{\mu\nu}$. Accordingly, for Einstein the most important achievement of the GR was not the notorious 'geometrization of gravity' but "unification of gravity and inertia" via the metric tensor $g_{\mu\nu.}$

Between 1907 and 1911 he ingeniously applied the equivalence principle to derive several consequences of his yet to be formulated relativistic theory of gravitation.

Note that in the case considered Einstein follows the path of the SR. Indeed, the new theory invention begins with the crossbred object construction, i.e. with the insertion of mass-energy relation into the theory of gravity. One of the important SR consequences is the tenet of equivalence of mass and energy. However, according to Einstein,

> this result suggests the question whether energy also possesses heavy (gravitational) mass. A further question suggesting itself is whether the principle of relativity is limited to nonaccelerated moving systems. (Einstein, 1907, p. 254)

From the very beginning Einstein was looking for such a theory of gravitation that should embrace both the knowledge on gravitation and inertia represented by the classical mechanics and the knowledge on the structure of space and time embodied by the SR. However, the crossbred object insertion—the introduction of inertial and simultaneously gravitational mass—leads to penetration of SR methods into Newtonian theory of gravity and to reverse penetration of Newtonian gravity methods into the SR. As a result, both theories were radically rebuilt from within and the corresponding changes in both of them were set up.

The changes were epitomized in the peculiar sequences of crossbred models, the byproducts of the transformation performed.

On the one hand, an inevitable consequence of SR penetration into Newtonian theory of gravity turned out to be Nordström's and Abraham's scientific research programmes. On the other hand, no less inevitable, owing to the equivalence principle, was Newtonian theory's penetration into the SR that led to the sequence of Einstein's works on the relativity principle generalization and to spreading the principle not only on inertial systems of reference, but on the various accelerated systems as well. Einstein used the principle of equivalence in order to transform the knowledge not of classical mechanics only but the knowledge embodied in *both*, classical mechanics and the SR. His theory of the static gravitational field, as well as his early attempts to generalize it, were nothing but a reinterpretation of the SR with the help of the introduction of accelerated frames of reference. His systematic treatment of such accelerated frames induced him to use generalized Gaussian coordinates in order to describe the coordinate systems adapted to these frames. It was then a natural step for him to consider the metric tensor. And with the introduction of the metric tensor Einstein constructed the theoretical object that was capable of representing gravitational and inertial theoretical objects on the same footing.

By the beginning of 1912, Einstein realized that he would ultimately have to proceed beyond a scalar theory of gravitation. His strategy was to move carefully in a step-by-step manner towards a full dynamical theory. The first step in the programme was to scrutinize the "gravito-static" case, the gravitational analogue of electrostatics. However, he was already thinking about the second step, the "gravito-stationary" case, the gravitational analogue of magnetostatics. His ultimate goal was to advance a theory for time-dependent gravitational fields.

In March 1912 he was able to inform Paul Ehrenfest:

> The investigations of gravitational statics (point mechanics, electromagnetism, gravitostatics) are complete and satisfy me very much. I really believe that I have found a *part of the truth*. Now I am considering the dynamical case, again also proceeding from the more special to the more general case. (quoted in Renn, 2007, p. 98)

As is well known, in 1908–1911 Einstein had neglected gravitation, possibly because of his preoccupation with the problem of quanta. But this, however, is only a part of the explanation. The remaining part consists in that he realized how much work had to be done to arrive at an ultimate global theory able to embrace all the particular results obtained, the "parts of the truth" as Einstein called them, transforming them into the details of a great edifice. And, since Einstein himself was delved into the peculiarities of the quanta, the problem of creating the gravitation global theory scaffolds had fallen on Abraham's and Nordström's broad shoulders.

However, one has to keep in mind that even the pathways of their theories' creation were outlined by Einstein himself, especially in his ground-breaking 1907 paper. Indeed, one of the important SR consequences states that $E = mc^2$. Since, in a gravitational field, the energy of a particle depends on the value of the gravitational potential at the position of the particle, the equivalence of energy and mass suggests that:

(1) either the particle's mass m;

(2) or the speed of light c (or both) must also be a function of the potential.

These possibilities, a dependence on the gravitational potential either of the speed of light c or of the inertial mass m, were later efficaciously explored by Max Abraham (1912a; 1912b; 1915) and Gunnar Nordström (1912; 1913a; 1913b), respectively.

And, first of all, it became clear that one can easily construct such a Lorentz-invariant theory of gravitation in which the inertial and gravitational masses are equal (Nordström, 1912–1914).

Besides, Einstein's static gravitational theory did not offer even a hint at how the global theory should be constructed. On the contrary, a Göttingen theoretician, a master of classical electrodynamics Max Abraham was one of the first scholars (along with Gustav Herglotz and Max Born) to propose that the four-dimensional line element, defining the infinitesimal distance between points in Minkowski space in terms of a constant metric, has to be replaced by a variable line element whose functional dependence of the coordinates is determined by a gravitational potential associated with the variable speed of light.

It was not accidental that Einstein turned to the global gravitational theory construction only *after* the publication of Abraham's first vector gravitational theory. It should be noted that for static fields Abraham's theory coincides with Einstein's. But the most valuable result of the hybrid theories of Nordström and

Abraham consisted in that the both theories maintained ***extremely promising hints*** on how the global theory could be created (Norton, 1992).

At first, by letting the geometry of Minkowski space depend on the gravitational potential (Abraham). At second, by representing the gravitational potential not by a single function but by a ten-component theoretical object on a par with the stress-energy tensor and having this tensor as its source (Laue and Nordström). At third, by including an effect of the gravitational potential on the measurement of space and time (Nordström).

The genesis of Einstein and Grossmann's '*Entwurf*'

Let me start from Nordström's trailblazing result derived with a help of Max von Laue's achievements. The result draws on the fundamental problem of classical electrodynamics—the problem of electron's electromagnetic mass that owes so much to Abraham's attainments (see, for instance, Abraham, 1909). If one computed total momentum and energy of the electromagnetic field of an electron, the result universally accepted at that time was:

(Total field momentum) = $4/3c^2$ (Total field energy) (Electron velocity)

Hence, as Poincaré and Einstein elucidated, there must be also stresses of a non-electromagnetic character within the electron ('Poincaré's stresses'). So, the puzzle Max von Laue addressed in 1911 was to find very general circumstances under which the dynamic of such an electron would agree with the relativistic dynamics of point masses. While Hermann Minkowski had introduced the four-dimensional stress-energy tensor in the course of elaboration of four-dimensional methods in SR, his use of the tensor was restricted to the special case of the electromagnetic field. Laue's work concentrated on extending the use of this tensor to the most general domain (Laue, 1911a; 1911b; 1911c). The properties of the tensor and its behavior under Lorentz transformations summarized a great deal of the then current knowledge of the behavior of stressed bodies.

As a result, Laue arrived at the expression for the stress-energy tensor $T_{\mu\nu}$ (μ,ν = 1,2,3,4) that embraced three main blocs.

(1) The first bloc represents the familiar three dimensional tensor p_{ik} (i,k = 1,2,3);

(2) The second bloc represents the momentum density **g** (g_x, g_y, g_z);

(3) The third bloc represents the energy flux **θ** (θ_x, θ_y, θ_z).

And, surely, the (T_{44}) component of the energy-momentum tensor represents energy. Einstein's equivalence principle prompted that each stress-tensor bloc should give *its own impact* into the gravitational field potentials, i.e. each bloc is related to the gravitational potentials of its own. Hence there should be a host of gravitational potentials—scalar ones, vector potentials, etc. and not a single one. That is why the overall gravitational field potential should be a group of several potentials and should in the most general case be described by a matrix, a tensor, since its components are transformed in the coordinate transformations like scalars, vectors, etc. The most pertinent analogy that played an important heuristic part was, of course, Maxwellian electrodynamics with 4-dimensional electromagnetic field potential $A^{\mu} = (\mathbf{A}, \varphi)$. The latter, in particular static electromagnetic field case, is reduced to static potential φ.

It is no wonder that in the 'Zurich Notebook', just before the '*Entwurf*' publication, Einstein had freely worked with tensors. The traces of the work can be easily found in his unpublished SR review, probably written between 1912 and 1914 for *Handbuch der Radiologie*. The heading of the Section 3, dealing with vectors, tensors, etc., speaks for itself: 'Some Concepts and Theorems of the Four-Dimensional Vector and **Tensor** Theories that Are Necessary for the Comprehension of Minkowski's Presentation of the Theory of Relativity' (see Klein *et al*., 1992, Doc. 1). Moreover, one of the subsections is promisingly entitled 'The Stress-Energy Tensor of Electromagnetic Processes'.

This feature was later diligently elicited by an eminent Göttingen specialist in electrodynamics Max Abraham in his 1915 thought-provoking paper 'Recent Theories of Gravitation'. The paper contained such an important passage critically analyzing Einstein's and Grossmann's '*Entwurf*' that it is worth quoting in full.

> The basic idea of the tensor theory of the gravitational field can be understood as follows. The energy density, which in a static field is determined by the divergence of the gradient of the gravitational potential, plays in the theory of relativity merely the role of one component of the resulting world tensor T; it is joined by nine other tensor components which characterize the energy flux and the stresses. The tensor theory assumes that, like the energy density (T_{44}), the remaining nine components $T_{\mu\nu}$ ($\mu, \nu = 1,2,3,4$) ***generate gravitational fields*** whose potentials $g_{\mu\nu}$ form a ten-tensor themselves (Abraham, 1915, p. 499).

The physical sense of the components is explained by Abraham below when he remarks that the integration of '*Entwurf*s' field equations

> is extraordinary difficult. Only the method of successive approximations promises success. In this one will usually take as a first approximation the solution that treats the field as static. Here, ***Einstein's theory becomes identical with Abraham's theory*** […]
>
> In his Vienna lecture A. Einstein takes the normal values of the $g_{\mu\nu}$ as the first approximation: $g_{11} = g_{22} = g_{33} = 1$; $g_{44} = -c^2$, $g_{\mu\nu} = 0$ for $\mu \neq \nu$; he considers the deviations $g^*_{\mu\nu}$ from these normal values as quantities of first order, and arrives, by neglecting quantities of second order, at the following differential equations: $\Box\, g^*_{\mu\nu} = T^m_{\mu\nu}$. For incoherent motions of masses, the last (T^m_{44}) among the components of the material tensor T^m is the most important; it determines the potential $g^*_{44} = \Phi^g$. Then follow the components T^m_{14}, T^m_{24}, T^m_{34}, which are of first order in v/c; these determine the potentials g^*_{14}, g^*_{24}, g^*_{34}, which can be viewed as the components of a space vector – (1/c) U^g. The remaining components of T^m are of second order in v/c. If one neglects quantities of this order, then one only needs to consider those four potentials, and obtains for them the differential equations
>
> $$\Box\, \Phi^g = c^2 \mu \qquad (60a)$$
>
> $$\Box\, U^g = c^2 \mu\, (v/c) \qquad (60b),$$
>
> where μ is the mass density [and $\Box$ is d'Alembert wave operator].
>
> Here the analogy with electrodynamics catches one's eye. Except for the sign, the field equations (60 a, b) agree with those that must be satisfied in the theory of electrons by the 'electromagnetic potentials', the scalar one (Φ) and the vector one (A). In this approximation, *the Einstein-Grossmann tensor theory of the gravitational fields leads to the same results as the vector theory sketched in* (IA) [i.e. the theory of Abraham]. (Abraham, 1915, pp. 500–501; my italics)

Abraham's conclusion can be re-affirmed by the consideration of Einstein's (1913c) paper 'On the present state of the problem of gravitation'. In this paper Einstein proceeded to show (on p. 1261) that the '*Entwurf*' theory reduced in suitable weak field approximation to a theory with a four-vector field potential that was formally analogous to Maxwellian electrodynamics. It was this approximation that yielded the weak field effects now commonly labeled as 'Machian'.

As his correspondence and the writings indicate, Einstein agreed with Nordström's assessment of the importance of Laue's work for gravitation theory. Moreover, some pieces of his 1912 and 1913 papers (his proposal to call T 'Laue's scalar', for instance) indicate that he had personal contacts with Laue and discussed the stress-tensor problems with him. Such personal communication is quite compatible with the fact that both Einstein and Laue were teaching in Zurich, with Einstein at the ETH and Laue at the University of Zurich (Norton, 2007). It should be added that the same year Nordström also came to Zurich where supposedly he had communicated with the both researchers.

It was instantly understood that in general the '*Entwurf*' equations are not covariant; they "remain covariant only with respect to linear orthogonal substitutions". Yet for a long time this peculiarity did not bother the authors. It testifies once more that the *Enwurf* field equations were born **not** from the covariance principle but represented a specific *generalization* of hybrid theories of Nordström and Abraham with a help of Laue's results. (The covariance principle appeared to be of great importance later—when the basic GR equations were derived in 1915).

Just as Einstein put it in his November 1913 letter to Paul Ehrenfest,

> The gravitational affair has been clarified to my *complete satisfaction* (namely the circumstance that the equations of the gr. field are covariant only with respect to linear transformations). For it can be proved that generally covariant equations that determine the field completely from the matter tensor cannot exist at all. Can there be anything more beautiful than this, that the necessary specialization follows from the conservation laws. (quoted in Klein *et al.*, 1993, Doc. 481; my italics)

However, on the other hand, according to later reminiscences,

> The equivalence principle allows us … to introduce non-linear coordinate transformations in such a [4-dimensional] space [with pseudo-Euclidean metric]; that is, non-Cartesian ('curvilinear') coordinates. The pseudo-Euclidean metric then takes the general form: $ds^2 = \sum g_{ik} dx_i dx_k$ summed over the indices i and k (from 1 to 4). These g_{ik} are then functions of the four coordinates, which according to the equivalence principle describe not only the metric but also the 'gravitational field'. (Einstein, quoted in Seelig, 1955, p. 55)

Certainly, the question was raised on obtaining the mathematical apparatus dealing with such whimsical mathematical objects. In particular, from the mathematical point of view, the task was to find a differential operator of second order for the metric tensor covariant with respect to the largest possible class of coordinate transformations. In August 1912, Einstein left Prague, where he had taught for a year and a half, to become a full professor at the *Eidgenössiche Technische Hochshule* (ETH). With Einstein's return to Zurich, he began a fruitful collaboration with his old friend Marcel Grossman. The collaboration ceased in 1914, when Einstein moved to Berlin to become a salaried member of the *Preussiche Akademie der Wissenschaften*.

Grossmann's help was needed to solve the problem. Grossmann dug out that the exquisite mathematical apparatus had contrived at the end of the 19th and the beginning of the 20th century by Riemann, Levi-Civita, Ricci, Christoffel *et al*. That is why the first part of the '*Entwurf*' containing the gravitational field equations was written by A. Einstein, and only the second one—by M. Grossmann (see Seelig, 1955, pp. 15–16 for details).

It is important for our study of the interrelations between Einstein, Nordström and Abraham's research programmes that the commencement of the metric programme should be related to 1912, when in a stupendous note published on 15 February 1912 as a reply to Einstein's critique, Abraham significantly rethought the lines with which he had earlier referred to Minkowski's formalism. He introduced instead an infinitesimal line element with variable metric, thus actually extending Minkowski's space-time to a more general semi-Riemann manifold.

> In lines 16, 17 of my note 'On the Theory of Gravitation' an oversight has to be corrected which was brought to my attention by a friendly note from Herr A. Einstein. One should read there: "we consider dx, dy, dz and du=icdt as components of a displacement $\overline{ds}$ in four-dimensional space". Hence
>
> $$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \qquad (12)$$
>
> is the square of the four-dimensional line element where the speed of light is determined by the equation (6) [i.e. $C^2/2 - (\mathrm{Co})^2/2 = \Phi - \Phi_0$]. (Abraham, 1912d, p. 1056)

In this way, Abraham was the first to contrive the mathematical representation of the gravitational potential that was to be at the heart of the GR, the general 4-dimensional line element involving a variable metric tensor. Of course, for the time being, Abraham's expression remained as isolated mathematical stunt without profound physical meaning. Yet, in my view, it is this equation that was referred by Einstein in his 26 March letter to his close friend Michele Besso: "At first (for 14 days!) I too was completely "bluffed" by the beauty and simplicity of his [Abraham's] formula" (quoted in Klein *et al.*, 1993, Doc. 377, pp. 436–437).

Driven by Abraham's stubborn persistence, Einstein in May 1912 thus finally recognized that a generalized line element $ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$, as suggested by Abraham's ingenious comment of three months earlier, represents the key to a generally relativistic gravitation theory.

Indeed, when Einstein had completed the elaboration of his theory of a static gravitational field (Einstein, 1912a), he took notice of the fact that his expression for the motion in a static gravitational field takes an *exactly* the same form as that given by Max Planck for a gravitation free Minkowski space, the only difference being that the speed of light is now assumed to be variable:

$$\delta \int ds = \delta \{ \int \sqrt{(-dx^2 - dy^2 - dz^2 + c^2 dt^2)} \} = 0 \qquad (1)$$

where δ stands for the variation of the subsequent integral and c = c(x, y, z). Einstein was now completely fascinated by the possibility the equation (1) offered for a generalization beyond the special case of static gravitational fields, as was indicated by him in an eminent 'Note in proof':

> This shows too—as has been shown for the ordinary theory of relativity by Planck—that the equations of analytical mechanics have a significant reading *far beyond* Newtonian mechanics. The Hamiltonian equation, which was the last one written down, gives an idea about how the equations of motion of the material point in a *dynamic* gravitational field are constructed. (Einstein, 1912a, p. 120)

Now the pertinent question became what happens when one considers the motion of point particles in the presence of *general*, i.e. *non-static* gravitational fields? In that case the abovementioned Laue's results amended by the 'equivalence principle' turned out to be of special importance. And in Section 2 of the '*Entwurf*' Einstein takes the abovementioned variation principle as a starting point to argue that for non-static gravitational fields, too, one should expect equation (1) to exhibit the motions of point-particles. The only difference is that now the line element on the left-hand side of the equation has to be that defined by a general metric tensor $g_{\mu\nu}$. This was the first time the metric tensor was introduced in a published paper. Three months after the '*Entwurf*', Einstein submitted a paper to the 85th conference of the German Society for Scientists and Physicians. In this paper he explicitly maintained that "A free mass point moves in a straight and uniform line according to Eq. (1), where $ds^2 = \sum g_{ik} dx_i dx_k$". And finally, on September 1913 in Vienna, Einstein presented a lecture eliciting the physical foundations of the '*Entwurf*' and those aforementioned general conditions (1)–(4) which any relativistic theory of gravity should satisfy (Einstein, 1913c).

As a result, the main attainment of the second streak consisted in the invention of metric tensor; the latter appeared to be a crossbred object that unified two different research traditions—a 'physical' tradition (scalar and vector theories of Einstein, Nordström and Abraham) and a 'mathematical' one (geometrical results of Riemann, Christoffel, Levi-Civita *et al.*). Now the components of g_{ij} had a *dual* function: on the one hand, they represented the physical gravitational potentials and on the other they were coefficients in the expression of $ds^2 = \sum g_{ij} dx^i dx^j$. By means of contrivance of the crossbred object g_{ij} the interpenetration of geometry and physics began: physics became geometrized, and geometry was made empirical (Zahar, 1989, p. 267).

Einstein himself, withal, did not consider the geometrization of the gravitational field as a major achievement of his research programme, stressing that the GR was no more and no less geometrical than Maxwell's theory of electromagnetism (see Lehmkuhl, 2014 for details).

The contrivance of ultimate 1915 November relativistic theory of gravitation

Moreover, the incessant interpenetration of geometry and physics eventually resulted in the construction of the GR fundamental theoretical scheme. The first stage of the interpenetration was concluded by the gravitational field equations of the '*Entwurf*': $R_{\mu\nu} = \varkappa T_{\mu\nu}$ with their ultimately simple premises of gravitational potentials being common partial derivatives of metric. However the further interplay of physics and geometry led to skillful modification of the plain scheme.

In a series of four publications from November 1915, submitted to the Prussian Academy of Science on the 4th, 11th, 18th and 25th, Einstein gradually replaced the '*Entwurf*' with a full-blooded metric theory of gravitation, solving incidentally the problem of Mercury's perihelion. To comprehend Einstein's reasoning in passing from the '*Entwurf*' to the GR one has to address the abovementioned Einstein's synthetic dual strategy that embraced and successfully reconciled both a physical and mathematical approaches to finding the ultimate field equations.

The '***dual method'*** suggested that Einstein "inductively" proceeded first and foremost from a set of the following constraints of a *physical* nature:

(P1) *The Newtonian limit*. Any relativistic theory of gravitation must obviously correspond to Newtonian theory in the case of weak and static gravitational fields. So, in this limit one must recover a Poisson-like equation for a scalar potential from the field equations.

(P2*) Conservation of energy and momenta*.

On the other hand, the complementary 'top-down' (van Dongen) or 'deductive' *mathematical* approach originated from the following two requirements.

(M1) *The Principle of Equivalence*. Observations made in a uniformly accelerated system are equivalent (in the abovementioned sense) to observations made in a homogeneous gravitational field.

(M2) *Generalization of the Principle of Relativity*. The laws of physics are identical for relatively inertially moving observers to observers in accelerated motion.

The (M1+M2) jointly imply that the field equations have to be generally covariant. Thus, a natural starting point for the mathematical approach would be to scrutinize a generally covariant mathematical abstract object that was known from the mathematics literature.

The key point is that the 'dual strategy' necessarily employed *both* physical and mathematical approaches: "it was an *iterative* process that began with trying out one approach, and then checking results with the demands or results of the other approach" (van Dongen, 2011, p. 11). Yet the physical approach first prevailed and lead to the '*Entwurf*'; but then the mathematics approach took over and incessantly led to 1915 November General Relativity.

It is interesting to note that in the 'Zurich Notebook' Einstein already had at his disposal the gravitational field equations that he would publish in November 1915. He had been necessarily led to them by the abovementioned 'interplay' of the dual method; however, they appeared to be inconsistent with a coherent system of 'physical' constrains. First and foremost, they did not provide the Newton metric $g_{\mu\nu}$ = diag (g_{00}, -1.-1,-1) in the weak field approximation. On the contrary, an obvious advantage of the '*Entwurf*' equations was that, when taking the Newtonian limit, one was not obliged to insert the notorious 'harmonic coordinates' restrictions to recover the Poisson equations in its familiar to Einstein form, and blatant inconsistencies with the Newton metric $g_{\mu\nu}$ = diag (g_{00}, -1.-1,-1) were therefore not to be expected.

Moreover, Einstein's confidence in the '*Entwurf*' was strengthened by a notorious 'hole argument' ("happenings in the gravitational field cannot be uniquely determined by generally covariant differential equations for the gravitational field"). So, eventually in November 1913 Einstein was able to confess to his former assistant Ludwig Hopf:

> I am now very satisfied with the gravitation theory. The fact that the gravitational equations are not generally covariant, something that quite disturbed me for a while, is unavoidable; it can easily be shown that a theory with generally covariant field equations cannot exist if the demand is made that the field is mathematically completely determined by matter. (2 November 1913, A. Einstein to L. Hopf; quoted in van Dongen, 2010, p. 22)

Yet, in the light of the dual strategy and the synthetic character that a true relativistic theory of gravity should possess it is no wonder that the absence of generally covariant field equations was strongly criticized by Max Abraham (1914, p. 8) and Gustav Mie (1914a; 1914b, p. 176). However, initially Einstein was stubbornly blind to such criticism.

Nevertheless, A. Einstein and M. Besso applied the metric $g_{\mu\nu}$ = diag (g_{00}, -1.-1,-1) to calculate for the first time the anomalous perihelion shift of Mercury. The result disappointingly appeared to be a factor of 2,4 off from the observed value.

Much distressed, Einstein would soon give up on the troubled theory; yet eventually his reasons turned out to be *threefold*. As he later confessed to Arnold Sommerfeld,

> [First] I proved that the gravitational field on a uniformly rotating system does not satisfy the gravitational field equations.
>
> [Second] the motion of the perihelion of Mercury came out as 18" instead of 45" per century.

> [Third] the covariance argument in my paper of last year does not give the Hamiltonian function H. When suitably generalized it allows an arbitrary H (Einstein to A. Sommerfeld, 28 November 1915, Doc. 153; quoted in van Dongen, 2010, p. 26).

As a result, Einstein found a promising way out of the '*Entwurf*' predicament: he returned to the half-forgotten mathematical strategy and to the requirement of general covariance that he harshly abandoned in the 'Zurich Notebook'. Subsequently, in the week after 11 November 1915, Einstein resolutely altered the calculation of Mercury's perihelion shift. He realized at last that covariant equations $R_{\mu\nu}$ - (R/2) $g_{\mu\nu}$ = k$T_{\mu\nu}$ cannot be consistent with the Newton metric $g_{\mu\nu}$ = diag (g_{00}, -1.-1,-1). Now he was forced to recognize the substantially tensor character of the theory and had to allow the spatial components of the metric tensor to be functions of the coordinates too: $g_{\mu\nu}$ = diag {g_{00}(x), g_{11}(x), g_{22}(x), g_{33}(x)}.The ds^2 equation that Einstein applied in the November 1915 paper to account for Mercury's motion strongly resembles the Schwarzchild black hole metric with its g_{00} = (1-2m/r) and the g_{11}-g_{33} terms depending on r and m.

Digging still deeper into the mathematical background, Einstein elicited, at last, the correct value of the perihelion shift: he famously found 43", which confirmed his desired 45"± 5" per century.

Thus, it appeared to be the reversion to the mathematical requirement of general covariance that finally brought salvation and that eventually relieved Einstein of his prejudices regarding the Newtonian limit $g_{\mu\nu}$ = diag (g_{00}, -1.-1,-1). He eloquently expressed his amazement at this truly 'enormous achievement' in a famous letter to Michele Besso:

> Read the articles! They bring the final salvation to misery. The most joyful aspect is *the accordance between the perihelion motion and general covariance*, the most striking however the circumstance that Newton's theory for the field is already incorrect at the first order of the equation (the terms g_{11}-g_{33} arise). The simplicity of Newton's theory is only due to the fact that the g_{11}-g_{33} do not arise in the first approximation to the equations of motion for a point mass" (Einstein to M. Besso, 21 December 1915, quoted in van Dongen, 2010, p. 29; my italics).

In the light of the proposed rational reconstruction of the GR genesis the abovementioned "accordance between the perihelion motion and general covariance" appears to be obvious. Einstein could arrive at correct explanation of the abnormal perihelion motion (grounded on the understanding that there are many different *gravitational* forces that act not only along the plane of Mercury rotation round the Sun) only when he had fully realized the genuinely ***tensor*** character of the gravitational field. That is, when he comprehended that the $g_{\mu\nu}$ is a host not only of scalar components like $g_{00}(r, m)$, but of the g_{11},g_{22},g_{33} components as well, each of which also depends on r and m.

It is important to acknowledge the following. The ultimate GR equations were posited as such only in November 1915, so the whole two years had passed after the blatant discrepancy between the '*Entwurf*' and the perihelion motion was revealed in 1913. And though Einstein indeed mentioned this discrepancy as one of his basic motivations for the '*Entwurf*–GR transition' (see the letter to Arnold Sommerfeld, 28 November 1915, Doc. 153, pp. 206–209 in Schulmann *et al.*, 1998), the things were not that simple. The '*Entwurf*' really could not explain *all* the abnormal motion, but Einstein quite reasonably did not take this fact as an ultimate refutation of his artful theory since the remaining unexplained parts of the precession could probably be due to other, non-gravitational factors. Einstein and Besso had found the faulty value already in June 1913, but during the two years—up to September 1915—Einstein constantly and persistently expressed confidence in the '*Entwurf*' theory. (See, for instance, his letter to Michele Besso from 10 March 1914, Doc. 514, pp. 603–604, Klein *et al.*, 1993). On my view, the main thrust in passing from the '*Entwurf*' to the GR came from his abovementioned dual strategy inducing him to *reconcile* physical and mathematical approaches. And only *after* he efficiently reconciled them, that is, only after he realized the general covariance of the gravitational field equations, could he turn to correct explanation of the Mercury perihelion. Hence the decisive impetus in constructing the General Relativity came from the attempts to reconcile physical and mathematical approaches, embodied in Abraham, Nordström and Einstein's crossbred theoretical models. Trustworthy explanation of Mercury perihelion motion appeared to be a lucky by-product of strenuous reconciliation efforts.

Conclusions

To recapitulate, the reconstruction proposed enables to highlight some discernible hallmarks of the process of the GR genesis that are commonly obfuscated by the rival approaches and to arrive at a more comprehensive account of the Einsteinian revolution *Inter-Theoretical Context*. (One should always keep in mind that real creative science is always messier and more complicated than philosophers of science like to fancy).

They are these hallmarks that allow to comprehend the true reasons for Einstein's victory over the rival programmes of Nordström and Abraham. The '*Entwurf*' theory and the GR "incorporated" Nordström's and Abraham's theories in a higher level theory. Thus, the true reason for Einstein's victory over the rival programmes does not consist in that he resolutely rejected the alternative theories. On the contrary, it consists in that he subsumed them all into a new framework. Hence the decisive drive in constructing the General Relativity came from the attempts to reconcile physical and mathematical approaches, embodied in the Abraham, Nordström and Einstein's crossbred theoretical models. Trustworthy explanation of Mercury perihelion motion appeared to be a natural by-product of strenuous reconciliation efforts.

The focus on the history of GR brings to mind the following vital question: what can one learn from Einstein's startling 1915 discovery about how science works?

In my judgement, the abovementioned story can better elicit the subtle interconnections between the two common accounts of the scientific method in GR that are tightly connected with empiricist and rationalist traditions in epistemology.

According to the first one (see, for instance, Popper, 1963; Lakatos, 1970; Kuhn, 1961), theories, research programmes and paradigms are invented by theorists to comprehend the phenomena which experimentalists have previously discovered. When a 'paradigm' accounts for all the available experimental data, it is applauded as a 'success'. Yet sometimes a new experimental discovery is made which was not predicted by the paradigm, making it 'anomalous'. Then, a new paradigm is artfully invented (or a new 'research programme' is conjured up) to explain the anomaly. Needless to say, it must also account for all the 'old' observations and experiments. When the 'new' paradigm provides a correct description of *all* the experiments at hand, we consider it as a proper 'replacement' for the old one.

So far so good. Yet the problem with this fine and lucid empiricist epistemological model is that it does not apply to the history of general relativity. Almost all the typical general-relativistic phenomena were unknown in 1915 when Einstein had arrived at his startling theory. These include the expanding universe, black holes, naked singularities, gravitational waves and so on. Not only were these queer phenomena not yet fixed in 1915, most of them had not even been imagined.

Just to compare the problem situation with proverbial history of quantum theory. It has to be acknowledged that a majority of 'characteristic quantum phenomenon' (Smolin, 2015) were well known to physicists before quantum mechanics was formulated in 1925–1926.

Empiricists commonly point to the eminent Mercury's perihelion shift as a textbook kind of the anomaly that GR had successfully explained away. But the real problem with the perihelion is that only Einstein and a small circle of theoreticians connected with him conceded that the true comprehension of this phenomena needed a radical breakthrough with the classical research traditions. The phenomena was well-known and thoroughly discussed half a century before Einstein, and the majority of astronomers had maintained that this shift could be duly explained either by a new planet or by more reliable and fine calculations based on standard and entrenched classical premises.

Moreover, the blatant discrepancy between the '*Entwurf*' and the perihelion motion anomaly was found *already* in 1913. But Einstein quite reasonably did not consider this fact as an *ultimate refutation* of his theory since, as was mentioned above, the remaining unexplained parts of the precession could be due to other, non-gravitational factors.

So, the things were not that simple as the empiricists maintain. Mercury's perihelion motion was not taken as a murderous evidence against Newton's theory of gravitation by the bulk of the physics community. In contrast, what should have been quite clear to *any* physicist was that Newton's theory of gravitation blatantly contradicted Einstein's special theory of relativity. *It was this commonplace anomaly that ought to be first and foremost explained away after 1905.*

But why not concoct a field theory for gravity within the perspicuous framework of special theory of relativity? This was the route Einstein and several of his contemporaries including Abraham, Nordström *et al*. took; some of them took it even until Eddington's 1919 eminent observations of light bending by the Sun. Thus, the pertinent question is why did Einstein ignore this beaten track in favour of a truly dubious and risky game?

And here another myth—now a rationalist one—is implemented. Now Einstein is pictured not as Humean and Machian empiricist which "ferrets out" (Einstein's words) the "first principles" but as a romantic "lone genius" who follows exquisite mathematics to reveal the sublime edifice of his great theory. Note that no one was more responsible for inventing this myth than Einstein himself (see, for instance, his 1949 'Autobiography', pp. 88–89). The trouble with the second myth is that it is at odds with history of science; we have, quite luckily, the 'Zurich Notebook' to retort.

The standpoint exposed in this study is an attempt to find our own "coherentist" way between the Scylla of rude empiricism and the Charybdis of plain rationalism by positing the reconstruction that contains both valuable epistemological insights as momenta of a more subtle account. The problem situation that led Einstein to GR was created not by the Mercury perihelion anomaly but by the blatant cross-contradiction between the special relativity and Newton's theory of gravitation. Yet to resolve it, i.e. to construct a global theory, Einstein had to take into consideration all the results obtained in hybrid theories of Abraham, Nordström and of himself. The crux of his theory—the fundamental theoretical scheme of GR—is a specific generalization of the crossbred theoretical schemes of Abraham, Nordström and Einstein on the basis of the famous 'principle of equivalence' that states the equivalence of gravitational and inertial mass and hence stresses that there should not be a glaring chasm between gravity and inertia. Einstein's theoretical scheme was better than the schemes of Abraham and Nordström not because it was the "true one" and its rivals were simply and obviously "wrong" but because it embraced them all in rather modified forms.

Accordingly, the core of Einstein's modus operandi was set up by the so-called 'dual strategy' that embraced both a physical and mathematical approaches to finding the ultimate field equations. In finding his equations Einstein took into account the results of experiments and observations but for him they were first and foremost the criteria for checking the efficacy of his unificationist efforts: "the most joyful aspect is *the accordance* between the perihelion motion and general covariance*"* (Einstein, 1915, abovementioned letter to Michele Besso).

The idea that a new theory can be invented to challenge the entrenched paradigm, even in the absence of experimental anomalies, was famously elaborated by Paul Feyerabend (see, for instance, Feyerabend, 1975). He contended that this strategy can succeed when the alternative paradigm suggests novel interpretations of already known experimental results and when these novel interpretations can provide the performance of new experiments that distinguish it from the old paradigm. In this sense the present study can be taken as an elucidation of Feyerabendian ideas that explicate the conditions under which the creation of such an alternative is possible. GR could help to create an avalanche of observational data referring to cosmology and astrophysics since it managed to reconcile two leading research traditions of the beginning of the 20th century that of special relativity and Newton's theory of gravitation.

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